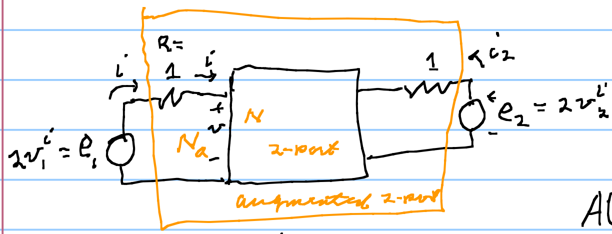
ENEE 610
09/17/20

Note Title 8/31/2020

Homework 2

S matrix properties, examples, ODE
realizations by Cs and Gvalues




$Y_a$

$e = 2v^i = v + i$ $\qquad v = v^i + v^r$

$2v^r = v - i$ $\qquad i = v^i - v^r$

$\underline{v^r = S v^i}$

$A v = B i$ for $N$

$A(v^i + v^r) = B(v^i - v^r)$

$(A - B)v^i = -(A + B)v^r$

$v^r = (B+A)^{-1}(B-A)v^i$

$S = (B+A)^{-1}(B-A)$

$e = v + i = v^r$

$i = Y_a e$

$v = e - i$

$= e - Y_a e$

$= (1_2 - Y_a)e$

$(1_2 - Y_a)i = (1_2 - Y_a)Y_a e = Y_a(1_2 - Y_a)e = Y_a V \Rightarrow \underset{A}{Y_a} v = \underset{B}{(1_2 - Y_a)} i$

$S = (1_2 - Y_a + Y_a)^{-1}(1_2 - Y_a - Y_a) = 1_2 - 2Y_a$ for $n$-ports $S = 1_n - 2Y_a$

$S = (B+A)^{-1}(B-A) = ([1_n + B^{-1}A)^{-1} B^{-1} B(1_n - B^{-1}A) = (1_n + Y)^{-1}(1_n - Y)$

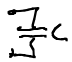
$\left(B(1_n + B^{-1}A)\right)^{-1}$

$= (1_n + B^{-1}A)^{-1} B^{-1}$

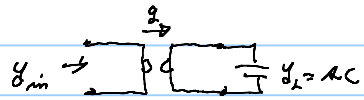
$B^{-1}A \Rightarrow A v = B i \Rightarrow B^{-1}A v = i = Y v$

$Y = B^{-1}A$

$S = 1_n - 2Y_a = (1_n + Y)^{-1}(1_n - Y) = (Z + 1_n)^{-1}(Z - 1_n)$

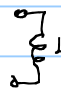
Ex:  $y_c = sC$, $S_c(s) = (1+sC)^{-1}(1-sC) = \frac{1-sC}{1+sC}$

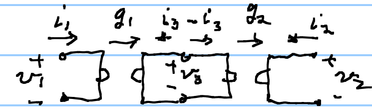
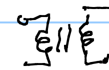
$S_c(j\omega) = \frac{1-j\omega C}{1+j\omega C}$ ; $|S_c(j\omega)| = \frac{\sqrt{1+(\omega C)^2}}{\sqrt{1+(\omega C)^2}} = 1$



$y_{in}(s) = \frac{y_{11}y_L + \Delta y}{y_{22}+y_L} = \frac{0+g^2}{sC} = \frac{1}{s(g^2/C)} = \frac{1}{sL}$

$Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$

$\Rightarrow \equiv$  $L = g^2/C$

 $\equiv$  $1 : T = g_1/g_2$

$i_1 = -g_1 v_3 \qquad -i_3 = -g_2 v_2$

$g_2 v_2 = i_3 = g_1 v_1 \qquad i_2 = g_2 v_3 = g_2\left(\frac{-i_1}{g_1}\right)$

$v_2 = T v_1 \qquad v_2 = \left(\frac{g_1}{g_2}\right) v_1 \qquad i_1 = \left(-g_1/g_2\right) i_2$

$i_1 = -T i_2$

$p(t) = (v_1 i_1 + v_2 i_2) = v^T(t)\, i(t) = \left(v_1 [-T] i_2 + (T v_1) i_2\right) = 0$ lossless in time

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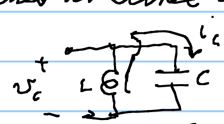
ODE in circuits

$\ddot{x} + \epsilon(x^2 - r^2)\dot{x} + \omega_0^2 x = 0$ , $x(0), \dot{x}(0)$ Van der Pol oscillator

$\dot{x} = y$

$\dot{y} = \ddot{x} = -\epsilon(x^2 - r^2)y - \omega_0^2 x$ $\Big\}$ state variable equations

if $\epsilon = 0$ this is like an LC parallel circuit



$i_L = -i_C$ , $v_L = v_C = sL(-i_L) = sL i_C$

$i_C = sC v_C$.

$L \dot{i}_C = v_C \qquad \Rightarrow \qquad \dot{i}_C = (1/L) v_C \Rightarrow \dot{x}_1 = \frac{1}{L} x_2$

$i_C = C \dot{v}_C \qquad\qquad \dot{v}_C = (1/C) i_C \qquad \dot{x}_2 = \frac{1}{C} x_1$